

## The Effect of Electromagnetic Polarization on the Performance of Adaptive Array Antennas

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**Abstract:** The polarized signals impinging on adaptive antenna arrays have significant impact on their performance. In this paper an investigation of the effect of polarized signals (desired and interference) on the performance of uniformly spaced steered beam adaptive array antennas is conducted, and a comparison between adaptive arrays with single dipole and cross-dipole elements is presented to show the effect of polarization on both of them. It is shown that the cross-dipole array antennas have better performance than the single dipole antenna array antennas if the polarization of the desired signal is unknown, while the single dipole arrays give better performance if the polarization of the desired signal is known.

### I. Signal Model

We assume that the elements of the array are dipoles separated by uniform distances  $\Delta z$  as shown in Fig. 1, where  $\theta$  and  $\phi$  denote standard polar angles.

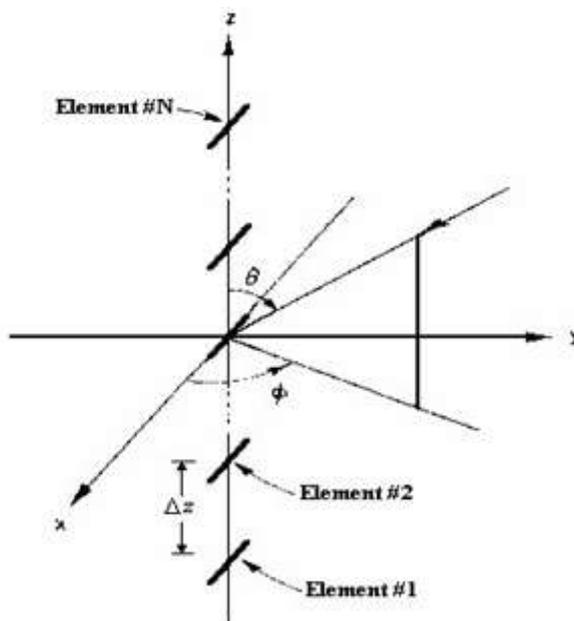


Fig.1 Geometry of single-dipole array.

A desired signal and  $M$  interference signals are incident on the array, where the desired signal arrives from angular direction  $(\theta_d, \phi_d)$  and the  $i^{\text{th}}$  interference signal from  $(\theta_i, \phi_i)$ . The signal vector of the array  $X$  can be expressed as:

$$X = [\bar{x}_1(t), \bar{x}_2(t), \bar{x}_3(t), \dots, \bar{x}_N(t)] = S_d + \sum_{i=1}^M S_{I_i} + S_n \quad (1)$$

where the desired,  $i^{\text{th}}$  interference and noise signal vectors are respectively defined as  $S_d \triangleq [s_{d_1}(t), s_{d_2}(t), \dots, s_{d_N}(t)]$ ,  $S_{I_i} \triangleq [s_{I_{i1}}(t), s_{I_{i2}}(t), \dots, s_{I_{iN}}(t)]$ , and  $S_n \triangleq [s_{n_1}(t), s_{n_2}(t), \dots, s_{n_N}(t)]$ . Here, it is assumed that each signal has an arbitrary electro-magnetic polarization. To characterize the polarization of each signal we make the following definitions.

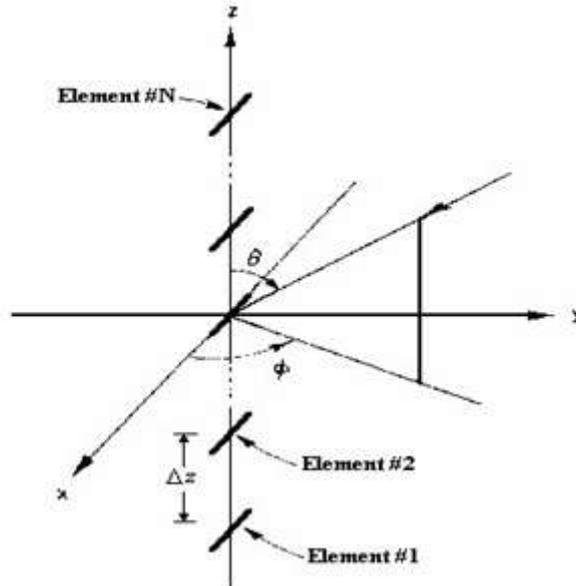


Fig.2 Geometry of cross-dipole array

Consider a transverse electromagnetic (TEM) wave propagating onto the array. We consider the polarization ellipse produced by the transverse electric field as we view the incoming wave from the coordinate origin shown in Fig. 1. Note that unit vectors,  $\hat{\phi}, \hat{\theta}, -\hat{r}$  in that order, form a right-handed coordinate system for an incoming wave. Since the electric field has transverse components [2],[3] then it may be represented by:

$$\vec{E} = E_{\phi} \hat{\phi} + E_{\theta} \hat{\theta} \tag{2}$$

where  $E_{\phi}$  is the horizontal component and  $E_{\theta}$  is the vertical component of the electric field.

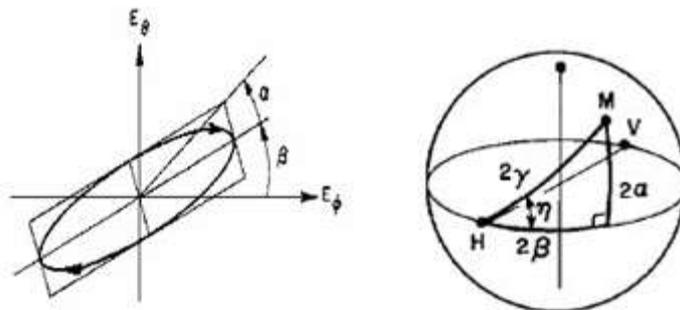


Fig.3(a) Polarization ellipse. (b) Poincare sphere

In general, as time progresses  $E_{\phi}$  and  $E_{\theta}$  will describe a polarization ellipse as shown in Fig. 3a. Given this ellipse, we define  $\beta$  to be the orientation angle of the major axis of the ellipse with respect to  $E_{\phi}$ , as shown in Fig. 3a. To eliminate ambiguities we define  $\beta$  to be in the range  $0 \leq \beta < \pi$  and the ellipticity angle  $\alpha$  to have a magnitude given by [2],[3].

$$\alpha = \tan^{-1} A_r \tag{3}$$

where  $A_r$  is the axial ratio defined as

$$A_r = \frac{\text{minor axis}}{\text{major axis}} \tag{4}$$

When the electric vector rotates clockwise  $\alpha$  is defined positive and when it rotates counter-clockwise  $\alpha$  is defined negative (when the incoming wave is viewed from the coordinate origin, as in Fig. 3a. The angle  $\alpha$  is always in the range  $-\pi/4 \leq \alpha < \pi/4$ . Fig. 3a depicts a situation in which  $\alpha$  is positive.

The electric field components for a given state of polarization, specified by  $\alpha$  and  $\beta$ , are given by (aside from a common phase factor)

$$E_{\phi} = A \cos \gamma \tag{5a}$$

$$E_{\theta} = A \sin \gamma e^{j\eta} \tag{5b}$$

where  $\eta$  and  $\gamma$  are related to  $\alpha$  and  $\beta$  by

$$\cos 2\gamma = \cos 2\alpha \cdot \cos 2\beta \tag{6a}$$

$$\tan \eta = \tan 2\alpha \cdot \csc 2\beta \tag{6b}$$

The relationship among the four angular variables  $\alpha, \beta, \gamma$  and  $\eta$  is most easily visualized by making use of the Poincare sphere concept [2]. This technique represents the state of polarization by a point on a sphere, such as point  $M$  in Fig.3b. For a given  $M$ ,  $2\gamma$ ,  $2\beta$ , and  $2\alpha$  form the sides of a right spherical triangle, as shown.  $2\gamma$  is the side of the triangle between  $M$  and a point labeled  $H$  in the figure;  $H$  is the point representing horizontal linear polarization. Side  $2\beta$  extends along the equator and side  $2\alpha$  is vertical, i.e., perpendicular to side  $2\beta$ . The angle  $\eta$  in Eq.5 and Eq.6 is the angle between sides  $2\gamma$  and  $2\beta$ . The special case when  $\alpha = 0$  in Eq.3 and Fig. 3b corresponds to linear polarization; in this case the point  $M$  lies on the equator. If in addition,  $\beta = 0$ , only  $E_\phi$  is nonzero and the wave is horizontally polarized. This case defines the point  $H$  in Fig. 3b[2]. If instead  $\beta = \pi/2$ , only  $E_\theta$  is nonzero and the wave is vertically polarized. Point  $M$  then lies on the equator at point  $V$ , diametrically behind  $H$ . The poles of the sphere correspond to circular polarization ( $\alpha \pm 45^\circ$ ), with clockwise circular polarization ( $\alpha = +45^\circ$ ) at the upper pole.

Thus an arbitrary plane wave coming into the array may be characterized by four angular parameters and amplitude. For example the desired signal will be characterized by its arrival angle  $(\theta_d, \phi_d)$ , its polarization ellipticity angle  $\alpha_d$  and orientation angle  $\beta_d$  and its amplitude  $A_d$  (i.e.,  $A_d$  is the value of  $A$  in Eq.5 for the desired signal). We will say the desired signal is defined by the parameters  $(\theta_d, \phi_d, \alpha_d, \beta_d, A_d)$ . Similarly, the  $i^{th}$  interference signal is defined by the parameters  $(\theta_i, \phi_i, \alpha_i, \beta_i, A_i)$ .

We assume each dipole in the array is a short dipole, i.e., the output voltage from each dipole is proportional to the electric field component along the dipole. Therefore the vertical and horizontal dipole outputs in Fig.1 and Fig.2 will be proportional to the  $z$ - and  $x$ -components, respectively, of the electric field. An incoming signal, with arbitrary electric field components  $E_\phi$  and  $E_\theta$ , has  $x, y, z$  components:

$$\vec{E} = E_\phi \hat{\phi} + E_\theta \hat{\theta} = (E_\theta \cos \theta \cos \phi - E_\phi \sin \phi) \hat{x} + (E_\theta \cos \theta \sin \phi - E_\phi \cos \phi) \hat{y} - (E_\theta \sin \theta) \hat{z} \tag{7}$$

When  $E_\phi$  and  $E_\theta$  are expressed in terms of  $A, \gamma$ , and  $\eta$  as in Eq.4-5, the electric field components become [2]

$$\vec{E} = A[(\sin \gamma \cos \theta \cos \phi e^{j\eta} - \cos \gamma \sin \phi) \hat{x} + (\sin \gamma \cos \theta \cos \phi e^{j\eta} + \cos \gamma \cos \phi) \hat{y} - (\sin \gamma \sin \theta e^{j\eta}) \hat{z}] \tag{8}$$

Adding to this expression the time and space phase factors, we find that a narrowband CW signal incident on the array from the direction of its main beam ( $\theta = \theta_{max}, \phi = 90^\circ$ ) produces a signal vector in the array (defined in Eq.1) as follows:

$$X = U[1, e^{-j\beta \Delta z \sin \theta_{max}}, e^{-j2\beta \Delta z \sin \theta_{max}}, \dots, e^{-j(N-1)\beta \Delta z \sin \theta_{max}}] \tag{9}$$

where  $U$  is a complex quantity given by

$$U = A[(\sin \gamma \cos \theta \cos \phi e^{j\eta} - \cos \gamma \sin \phi) - (\sin \gamma \sin \theta e^{j\eta})] \cdot e^{j(\omega_c t + \psi)} \tag{9a}$$

and  $\beta = 2\pi/\lambda$  is the wave-number of the narrowband signal  $A_i$  is the centre frequency of the signal,  $\psi$  is the phase of the signal at the coordinate origin at  $t = 0$ . The output signal from the  $i^{th}$  element  $\tilde{x}_i(t)$ , which is assumed to be a complex random process, is multiplied by a complex weight  $w_i$  and summed with the other  $N - 1$  output signals to produce the array output  $\tilde{s}_o(t)$ . The steady-state weight vector which maximizes the output SINR as mentioned in the previous chapters is given by [1]:

$$w = [w_1, w_2, w_3, \dots, w_N]^T = [I_N + K\Phi]^{-1} \bar{w}_o \tag{10}$$

where,  $\Phi = E\{X^* X^T\}$  is the covariance matrix, and  $\bar{w}_o = [w_{10}, w_{20}, w_{30}, \dots, w_{N0}]$  is the steering vector of the array [1],  $I_N$  is the identity matrix,  $K$  is the feedback loop gain,  $T$  denotes transpose,  $*$  denotes complex conjugate, and  $E\{\cdot\}$  denotes the expectation. The output of the array for such a signal would be

$$\tilde{s}_o(t) = X^T w = U \cdot [w_1 + w_2 e^{-j\mu_1} + w_3 e^{-j\mu_2} + \dots w_N e^{-j\mu_{N-1}}] \cdot e^{j(\omega_c t + \psi)} \tag{11}$$

where,

$$\mu_i \triangleq i \Delta z \beta \cos \theta_{max} \tag{11a}$$

The quiescent pattern of the array will produce a maximum SINR output if

$$w_1 = w_2 e^{-j\mu_1} = w_3 e^{-j\mu_2} = \dots = w_N e^{-j\mu_{N-1}} \tag{12}$$

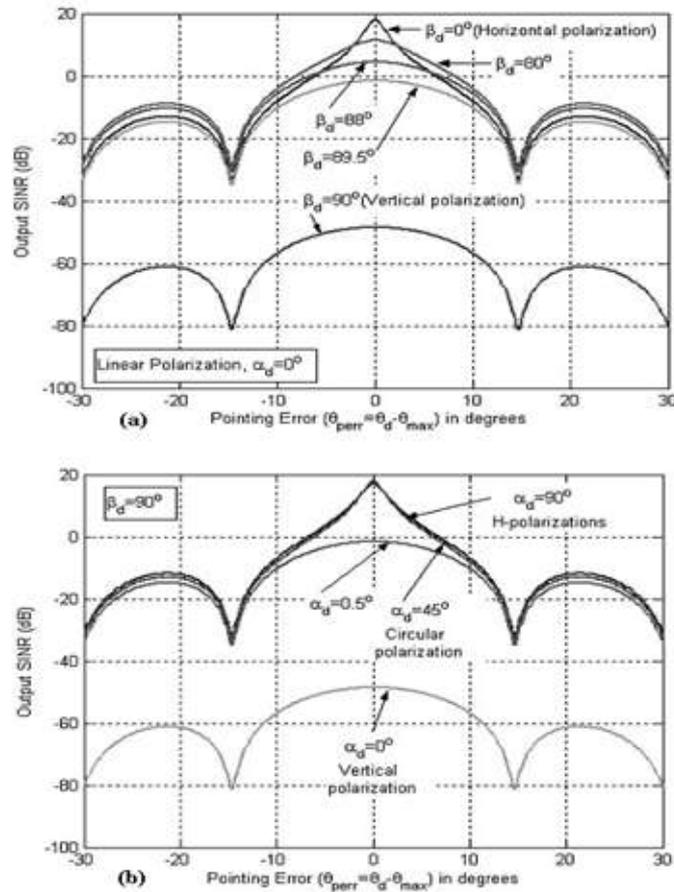
Therefore, for a given  $\theta_{max}$ ,  $\bar{w}_o$  is typically chosen as

$$\bar{w}_o = [e^{-j\mu_{N-1}}, \dots, e^{-j\mu_2}, e^{-j\mu_1}, 1]^T \tag{13}$$

Using this equation, the steady-state weight vector in Eq.10, could be evaluated.

## II. Simulation Results

Fig. 4 shows the output SINR of an 8-single dipole uniformly spaced antenna array (with  $\lambda = 0.5$ ) as a function of the pointing error relative to the desired signal ( $\theta_{perr} = \theta_d - \theta_{max}$ ), without interference.



**Fig.4** SINR vs. ( $\theta_{\text{perr}}$ ) for single-dipole array  $NR = 10\text{dB}$ ,  $\theta_d = 0^\circ$ . No interference. (a)  $\alpha_d = 0^\circ$  (b)  $\beta_d = 90^\circ$

It is assumed that a 10dB-SNR desired signal is incident on an array (shown in Fig.1) where the direction of the main beam is  $\theta = 90^\circ$ ,  $\phi = 90^\circ$ . Consider first that the polarization of the desired signal is linear (i.e.  $\alpha_d = 0^\circ$ ). It can be seen in Fig. 4a that the output SINR is maximum when the polarization of the desired signal is horizontal (i.e. along the x-axis) and  $\theta_d = 90^\circ$ , while it deteriorates rapidly as the pointing error increases, and when its polarization converts to vertical. Secondly, Fig. 4b shows the effect of pointing error on the output SINR for various types of desired signal polarizations.

It can be seen that changing from horizontal polarization to circular polarization has little impact on the performance of the array in terms of sensitivity to pointing errors. On the other hand, Fig. 5 shows the output SINR of the array as a function of the input SNR of the desired signal per element. It can be seen the horizontally polarized desired signal gives the best performance because the electric field  $\vec{E}$  lies along the x-axis (i.e. in parallel with the dipoles).

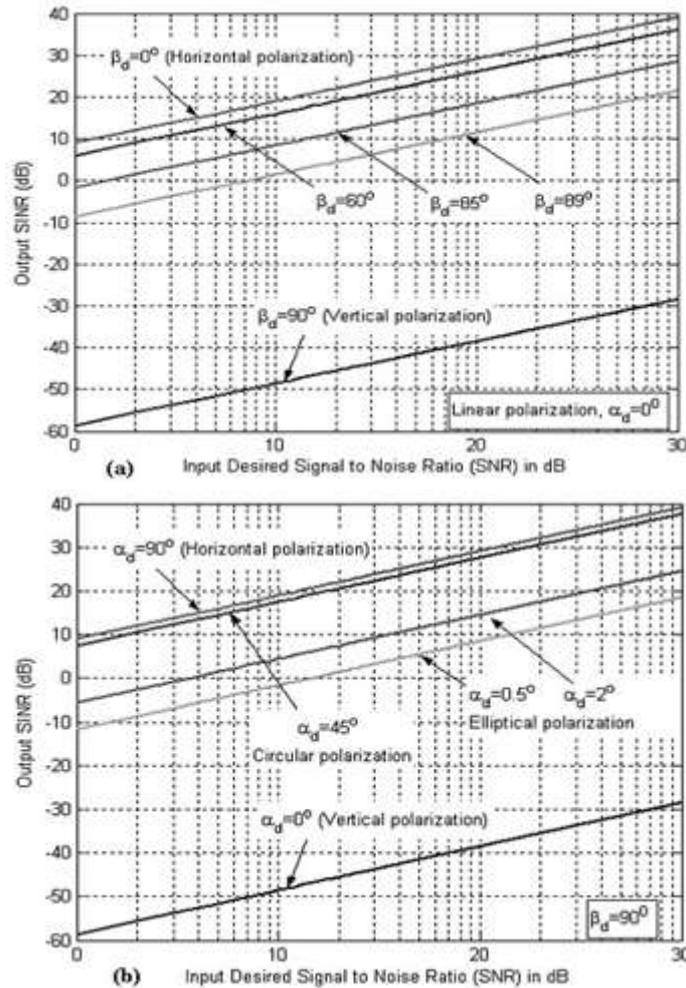


Fig.5:SINRvs.input SNR/elementforsingle-dipole array. Nointerference.(a)  $\alpha_d = 0^\circ$  (b) $\beta_d = 90^\circ$ .

Figure 5a shows the situation for linear polarization, while Figure5b shows the performance of the array for various types of polarizations (Vertical, Elliptical, Circular and Horizontal). Again it can be seen that the horizontal polarization case provides the best performance of the array.

In Fig. 6, it is assumed that a 30dB-SNR horizontally polarized desired signal with  $\theta_d = \theta_{max} = 90^\circ$ , and one 30dB-INR interference signal are incident on the array with  $\phi_d = 90^\circ, \phi_i = 90^\circ$ . Fig. 6a shows the performance of the array versus the relative DOA of the interference signal ( $\theta_i - \theta_{max}$ ), when the polarization of the interference signal is linear. The interference signal has no effect on the array when its polarization is vertical, while it has strong negative effect on the output of the array when its polarization is horizontal and when its DOA is  $\theta_i = 90^\circ$  (i.e. the same direction of the desired signal). The effect of various types of polarizations of the interference signal is shown in Fig. 6b.

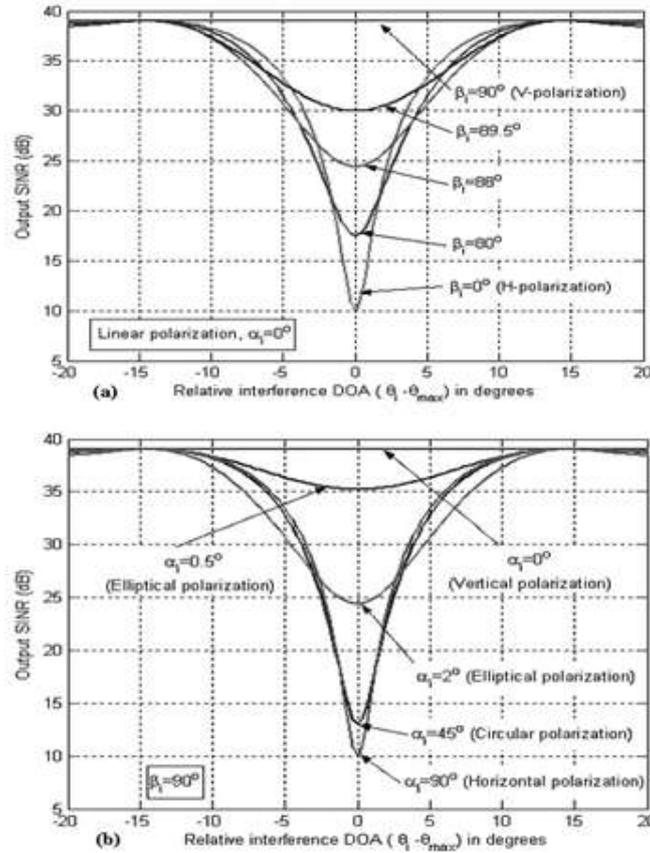


Fig.6: SINR vs. Interference DOA for single-dipole array. INR=SNR=30dB.  
 (a)  $\alpha_d = 0^\circ$  (b)  $\beta_d = 90^\circ$

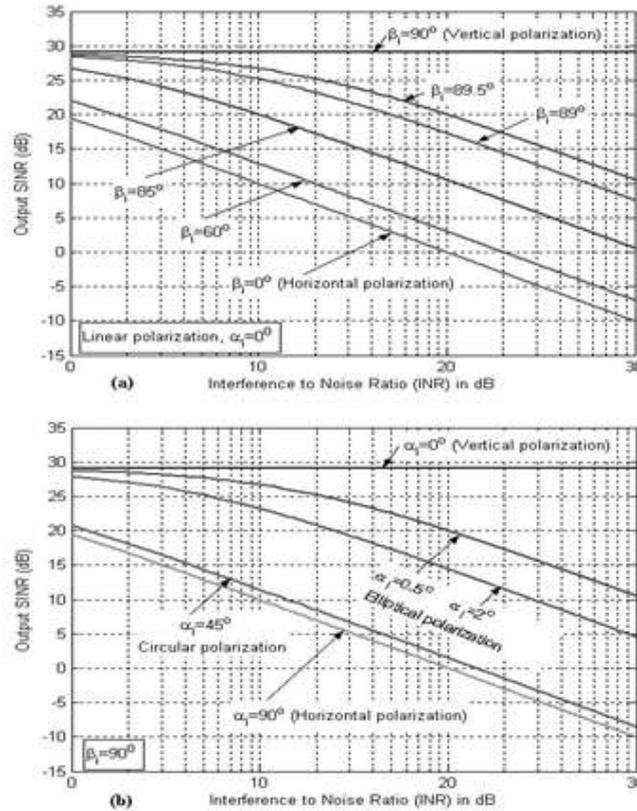


Fig.7: SINR vs. INR for single-dipole array. SNR=20dB.  $\theta_d = 0^\circ, \theta_i = 0^\circ$   
 (a)  $\alpha_d = 0^\circ$  (b)  $\beta_d = 90^\circ$

The output SINR as a function of the INR of the interference signal is shown in Fig. 7. Here, both the desired and interference signals have incident angles of  $\theta_d = \theta_i = 0^\circ$ , and the pointing error  $\theta_{perr} = (\theta_d - \theta_{max}) = 0^\circ$ . It can be seen that the vertically polarized interference signal has no effect on the array, while the horizontally polarized interference signal has the most negative effect on the array. In the latter case the output SINR decreases quickly as INR increases.

Fig. 7b shows the performance of the array when the polarization of the interference signal is vertical, elliptical, circular and horizontal. Fig. 6 and Fig. 7 show that the effect of the interference signal is minimized if its polarization does not match the arrangement of the dipoles, while the polarization of the desired signal does match the dipole arrangement. By comparing these results with those in Fig. 4 and Fig. 5 it can be concluded that if the polarization of the desired signal and its DOA is known, this information can be employed to get the best performance of the array.

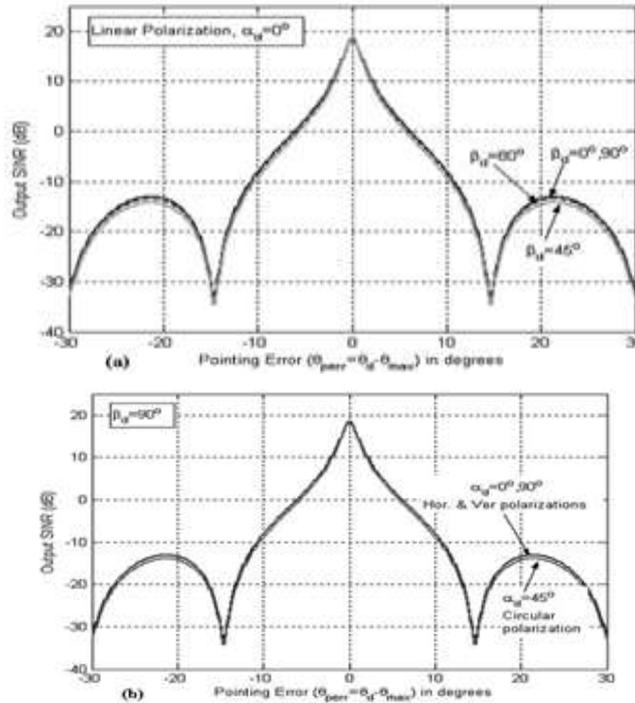


Fig.8: SINR vs.  $(\theta_{perr})$  for cross-dipole array SNR=10dB,  $\theta_d = 0^\circ$ . No interference. (a)  $\alpha_d = 0^\circ$  (b)  $\beta_d = 90^\circ$ .

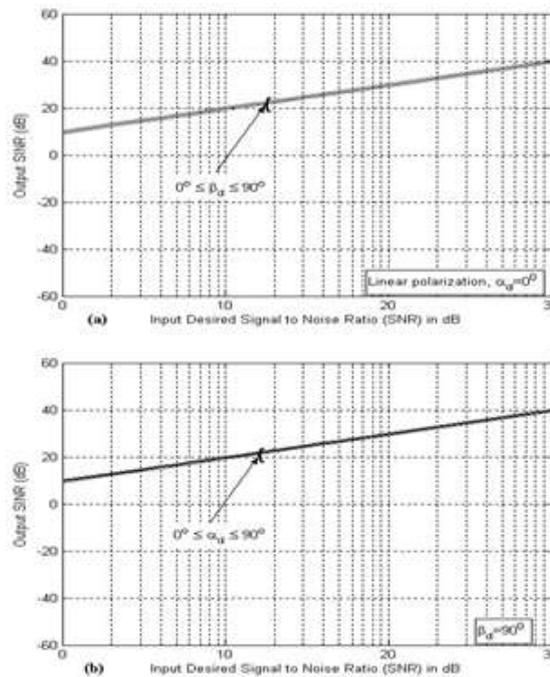


Fig. 9: SINR vs. input SNR/element for cross-dipole array. No interference. (a)  $\alpha_d = 0^\circ$  (b)  $\beta_d = 90^\circ$ .

However, if the polarization of the desired signal is completely unknown, cross-dipoles (shown in Fig. 2) can be used to enhance the performance of the array.

This arrangement makes the array performance independent of the polarization of the desired signal. Unfortunately this also makes the array performance independent of the polarization of the interference signal.

Fig. 8 shows the output SINR of eight element cross- dipole array of Fig 2, as a function of the pointing error relative to the DOA of the desired signal ( $\theta_{perr} = (\theta_d - \theta_{max})$ ). Here, it is assumed that  $\phi_d = 90^\circ$ , SNR=10dB and no interference exists. It can be seen that the polarization of the desired signal has approximately no effect on the performance of the cross- dipole array, in comparison with its effect on the performance of single dipole array previously illustrated in Fig. 4. This could be advantageous when the polarization of the desired signal is unknown.

Similarly, the cross-dipole array figures, (Fig. 9, Fig. 10, and Fig. 11) corresponding to single-dipole array figures (Fig. 5, Fig. 6, and Fig. 7, respectively), show that the effect of polarized desired signal is insignificant in the former case (the change in SINR is less than 2 dB). However, while the polarized interference signal maintains its negative effect on the cross-dipole array (as shown in Fig. 10 and Fig. 11), it could be mitigated in the single-dipole array if its electric field component is perpendicular to the dipoles of the array, as previously discussed in Fig. 6 and 7.

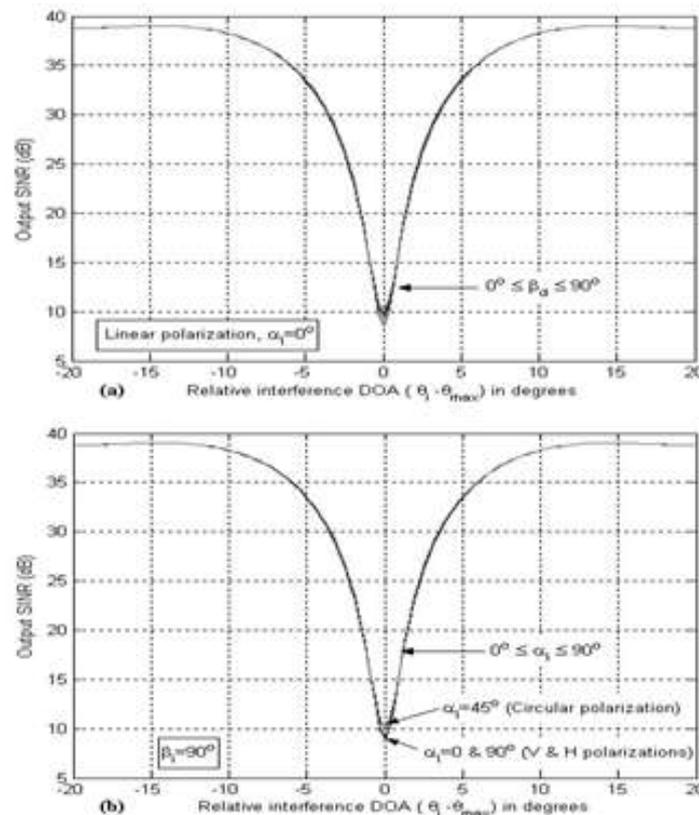


Fig. 10: SINR vs. Interference DOA for cross-dipole array. INR=SNR=30dB. (a)  $\alpha_d = 0^\circ$ (b) $\beta_d = 90^\circ$ .

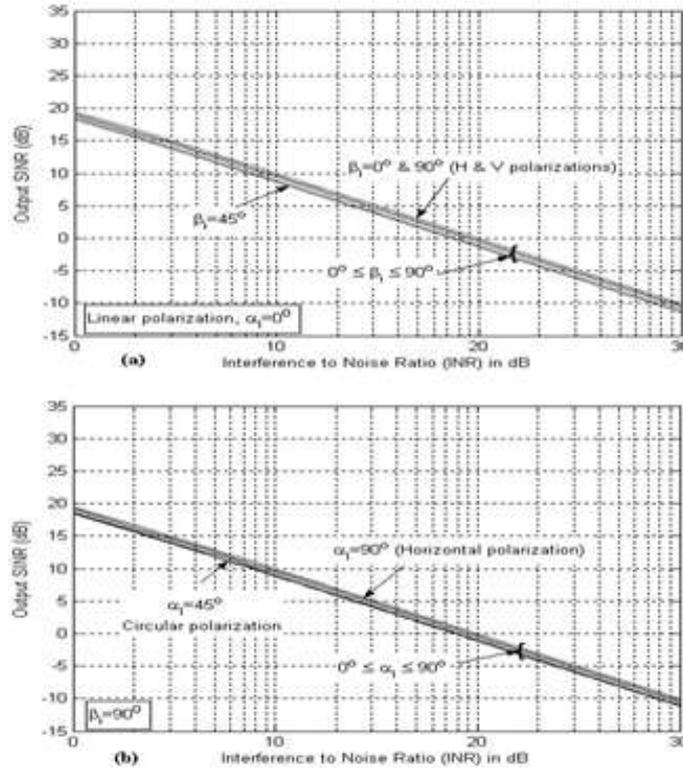


Fig. 11 SINR vs. INR for cross-dipole array, SNR=20dB,  $\theta_d = 0^\circ$ ,  $\theta_i = 0^\circ$   
 (a)  $\alpha_d = 0^\circ$  (b)  $\beta_d = 90^\circ$

### III. Conclusions

In this paper, we have presented the effect of polarized signals (desired and interference) on the performance of uniformly spaced steered beam adaptive array antennas. It is shown that the knowledge of the polarization of the desired signal and its DOA can be employed to get the best performance of the single dipole antenna array. However if the polarization of the desired signal is completely unknown, cross- dipoles can be used instead of single dipoles to enhance the performance of the array.

### References

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